In-depth analysis of the frequency stability of optically pumped cesium beam frequency standards

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***Abstract*— In the frame of an ESA Contract aiming at the feasibility study of a compact, very stable, optically pumped cesium beam clock for Galileo, we have completed our previous analysis of the frequency stability of such a frequency reference [1]. This analysis concerns the single optical frequency configuration. All possible noise sources are considered. Optimum operating conditions providing the best frequency stability are defined and presented. It is shown, and justified by simple arguments, that a resonator using a microwave cavity with a phase difference $\phi = \pi$ between the two oscillatory fields may either provide a better frequency stability than a cavity of the same length but with $\phi = 0$, or provide a given frequency stability with a cesium consumption significantly lower than in the $\phi = 0$ configuration. The analysis is applied to the OSCAR resonator conceived and operated at Observatoire de Neuchâtel, with two different operating conditions that provide very high performances : the theoretical and experimental results are in excellent agreement. It is concluded that the presented analysis can be safely used to design very high performance, compact, optically pumped cesium beam clocks.**

I. INTRODUCTION

In the frame of the feasibility study of a compact and very stable optically pumped cesium beam frequency standard for space navigation, we have completed our previous analysis of the frequency stability of such a frequency reference [1]. Indeed, in spite of many experimental works in the domain [2 - 7], no detailed analysis of the overall optimization of the clock was available : the present work tries to compensate for this lack in the case of the simplest possible structure using only one laser and one optical frequency, which is desirable for space applications. The single frequency configuration implies to chose a pumping transition : this choice will not be emphasized here.

Our approach consists in determining the optimum design and the optimum operating conditions of the atomic resonator with its associated electronics : all the noise sources including the power and frequency fluctuations in the microwave interrogation signal and in the optical signal, the two possible cavity designs and the main operating parameters are considered, in order to obtain the best short term and long term frequency stability with a given atomic flux from the cesium oven.

Transposing to the optical domain usual concepts of RF electronics, we define an effective noise figure of the detection of atoms, where the atom to photon conversion and the photon to electron conversion are taken into account. The possible additional sources of noise are considered and represented by a fictitious spurious flux of cesium atoms [1].

Then optimum operating conditions providing the best frequency stability are investigated. It appears that the optimum microwave level and the optimum modulation depth in the microwave signal depend on the ratio of the additional noise to the fundamental noise related to the atomic flux: in particular, these optimum parameters are quite different if the atomic resonator is operated at low flux or high flux for a given additional noise. The effect of the phase difference, 0 or π, between the two oscillatory fields in the atomic resonator has already been pointed out [1]. Here, the advantage given by the inverted fringe configuration is explained by simple physical arguments and calculated versus a parameter I_{bg} linked to the additional noise and to the useful atomic flux.

II. THEORETICAL NOISE BUDGET AND FIRST CONCLUSIONS

The fluctuations existing in the clock signal can be separated into three groups:

- noises related to the granular nature of atoms, photons and electrons. We call them "*intrinsic noises*" and their effect is directly linked to the useful atomic flux, i.e. that contributing to the detected signal
- *additional noises* independent of the atomic flux, of the laser frequency noise and of the microwave frequency noise
- other noise contributions from different origins : intermodulation effect, amplitude noise of the laser via the stray light…

A. Intrinsic noise : effective noise figure

The approach is based on the Noise Figure (NF) concept. The reference level corresponding to the minimum value is the atomic shot noise (NF = 1). The first NF to be considered is

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that of the atom to photon conversion. If the pumping transition is saturated, it is [8]

$$F = 2 - 1/\beta \qquad \text{with } \beta \geq 1 \tag{1}$$

In the detection zone, we denote I_0 the atomic flux that would be optically detected if the transition probability were equal to unity, I_1 the flux of atoms which have undergone the microwave transition ($I_1 = I_0 P$ where P is the transition probability), β the average number of photons per atom, γ the efficiency of the optical collection and η the quantum efficiency of the photo-detector. The electronic flux at the output of the detector is:

$$I_e = \beta\gamma\eta I_1 \tag{2}$$

The power spectral density (PSD) $S^{(1)}(I_e)$ of this flux is equal to the sum of the amplified initial PSD $S(I_p)$ and of the partition noise PSD related to the optical collection and to the photon-electron conversion [9].

$$S^{(1)}(I_e) = 2\,[\,F\,(\beta\gamma\eta)^2 + \beta\gamma\eta\,(1 - \gamma\eta)\,]\,I_1 \tag{3}$$

We can define an effective noise figure F^* of the opto-electronic detection including the atom-photon conversion, writing

$$S^{(1)}(I_e) = 2\,F^*\,(\beta\gamma\eta)^2\,I_1 \tag{4}$$

$$\text{with} \quad F^* = 2 - 2/\beta + 1/\beta\gamma\eta \tag{5}$$

We must mention that this expression of the effective noise figure F^* was also found by Pierre Thomann with a different approach [10]. Expression (5) may also be written as $F^* = 2 + 1/\beta\,(1/\gamma\eta - 2)$ which shows that F^* does not depend on β if $\gamma\eta = 0.5$. When $\gamma\eta < 0.5$, F^* decreases if β is increased, consequently a high value of β is preferable in practical cases.

B. Additional noises

We only consider white noise sources which are independent of the atomic flux, of the laser frequency noise and of the microwave signal frequency noise : they are i) the shot noise of the spurious light flux I_{sp} reaching the photo detector, ii) the thermal noise of the photo detector and of the trans-impedance amplifier, related to their resistances and iii) other possible additional noises related to the atomic resonator or to the signal processing, described by the PSD S_{extra} referred to the output of the photo-detector. The total PSD of these additional noises, referred to the output of the photo detector - electron flux - is given by

$$S_{add} = 2\,\eta\,I_{sp} + 4\,kT/(e^2 R_{sf}) + S_{extra} \tag{6}$$

where k is the Boltzmann constant, T is the absolute temperature of the photo-detector, R_{sf} is the resistance equivalent to the paralleled resistances of the photo-detector and of the trans-impedance, e the electron charge. The current noise of the operational amplifier is not considered since it is negligible in currently available devices.

C. Fictitious atomic flux representing the additional noises

Finally, the PSD of the ensemble of non correlated noises that disturb the observation of the resonance line is the sum of the PSDs of the intrinsic noise and of the additional noises. At the output of the photo detector, the total PSD of the electronic flux fluctuations is

$$S_{total} = 2\,F^*\,(\beta\gamma\eta)^2\,I_1 + 2\,\eta\,I_{sp} + 4\,kT/(e^2 R_{sf}) + S_{extra} \tag{7}$$

We can define a fictitious atomic flux I_b representing the additional noises [1] such as

$$S_{total} = 2\,F^*\,(\beta\gamma\eta)^2 (\,I_1 + I_b) \tag{8}$$

$$\text{with} \quad I_b = [2\,\eta\,I_{sp} + 4\,kT/(e^2 R_{sf}) + S_{extra}]\,/\,2\,F^*\,(\beta\gamma\eta)^2 \tag{9}$$

First conclusions : referred to the atomic flux level, i.e. without considering the amplification given by $\beta\gamma\eta$, (8) may be written as

$$S_{total}/(\beta\gamma\eta)^2 = 2\,F^*\,(\,I_1 + I_b) \tag{10}$$

which shows the requirements to meet in order to obtain the best frequency stability: i) a low value of the effective noise figure F^* in order to be as close as possible to the shot noise limit of the atomic beam, ii) a low value of S_{add} and iii) a high value of $F^*\,(\beta\gamma\eta)^2$ in order to properly reject the additional noises S_{add}. This implies to use an atomic transition with a high value of β (as high as possible), a light collection with a high efficiency γ, but with a low spurious light level and a photo-detector with a high quantum efficiency η.

D. Other noise sources : laser and microwave signals

It is usually considered that the effects of amplitude noise of the laser and of the microwave signal are quite negligible.

The direct effect of the laser amplitude noise in the clock signal is indeed negligible since the pumping transition is completely saturated, but a slight indirect effect may possibly be observed via the stray light, which is proportional to the laser intensity: in practise, it is verified that the stray light is shot noise limited [7], and consequently not significantly disturbed by the laser amplitude noise.

The automatic adjustment of the microwave amplitude at the optimum level cancels the effect of a possible microwave amplitude noise.

1) Laser frequency noise

In the state preparation zone, the laser frequency noise may induce slight fluctuations via the unpumped atoms. In practice, the rate of unpumped atoms may be low enough so that the associated additive noise is negligible [11]. At the detection level, the possible effect is the same.

2) Microwave frequency noise

Because of the frequency modulation–amplitude demodulation process necessary to lock the oscillator to the atomic resonance in passive standards, the frequency noise of the microwave interrogation signal, via an inter-modulation effect, generates very slow fluctuations in the error signal of the frequency servo and induces a limitation to the frequency stability [12] [13]. The resulting Allan standard deviation limit is given by

$$\sigma_y^{lim}(\tau) \approx [S_y(2f_M)]^{1/2}\,/\,2\;\tau^{-1/2} \tag{11}$$

where $S_y(2f_M)$ is the PSD of the relative frequency fluctuations of the microwave probe signal at Fourier frequency twice the modulation frequency f_M of the microwave probe signal. This noise source cannot be identified with an additional noise as defined previously. Its Allan variance must be directly added to the variance resulting from the intrinsic and additional noises. With a high quality ultra stable oscillator at 5MHz or 10MHz, $\sigma_y^{lim}(\tau = 1s)$ is in the range 1×10^{-13} to 3×10^{-13}, which is not a problem for an expected clock performance in the range 1×10^{-12} to 3×10^{-12} at 1s.

E. Expression of the frequency stability

The frequency stability is the relative frequency fluctuation of the oscillator slaved to the atomic reference, characterized for example by the Allan standard deviation $\sigma_y(\tau)$. The PSD of the frequency fluctuations of the oscillator's signal locked to the atomic resonance is equal to the PSD of the atomic response divided by the square of the slope of the resonance curve at the probing point. It may be written as

$$S(\delta\omega) = 2F^* \, [\, I_b + I_0 P(b,\omega)] / \, I_0^2 \, [\partial P(b,\omega) \, / \, \partial\omega]^2 \qquad (12)$$

where b is the microwave probe level expressed in rad.s^{-1}, ω is the angular frequency, $\omega = \omega_0 \pm \omega_m$, , with ω_0 the resonance frequency and ω_m the modulation depth, $P(b,\omega)$ and $\partial P(b,\omega)/\partial\omega$ are the transition probability and its derivative at the interrogation point. One shows that the resulting Allan standard deviation is

$$\sigma_y(\tau) = (\omega_0 \, T_0)^{-1} \, (2I_0)^{-1/2} \, (F^*)^{1/2} \quad H^{1/2} \, \tau^{-1/2} \qquad (13)$$

where T_0 is the time of flight of atoms between the arms of the cavity , length L, at the most probable velocity in the oven, and the dimensionless parameter H is defined [1] by

$$H = 2 \, [I_{bg} + P(b,\omega)] / \, [T_0^{-1} \, \partial P(b,\omega) \, / \, \partial\omega]^2 \qquad (14)$$

with $$I_{bg} = I_b \, / \, I_0 \qquad (15)$$

III. Optimum Operating Conditions and Structure of the Resonator

The atomic resonator uses a microwave cavity where the phase difference between the two oscillatory fields may be 0 or π, respectively providing a bright fringe or an inverted (dark) fringe [2]. In (14), H depends on three variable parameters : I_{bg}, b or rather $b\tau_0$, and $\omega_m T_0$. τ_0 is the atomic time of flight across one arm of the cavity, length l, at the most probable velocity in the oven. For a given value of I_{bg}, $H^{1/2}$ shows an absolute minimum which is the minimum minimorum, for particular values of $b\tau_0$ and of $\omega_m T_0$ that depend on I_{bg}. The variation of this optimum versus I_{bg} is shown in fig.1 : we observe a systematic advantage for the resonator providing an inverted fringe (cavity with $\phi = \pi$ instead of $\phi = 0$), and the relative advantage is larger for smaller values of I_{bg}, i.e. for a higher performance operation. This absolute minimum only concerns the short term frequency stability. Indeed, there is an other possible optimum, which is much more interesting for the general behaviour of the clock since it suppresses the effect of the cavity detuning [1] and the corresponding frequency fluctuations, related to the temperature for example, thus improving the long term stability.

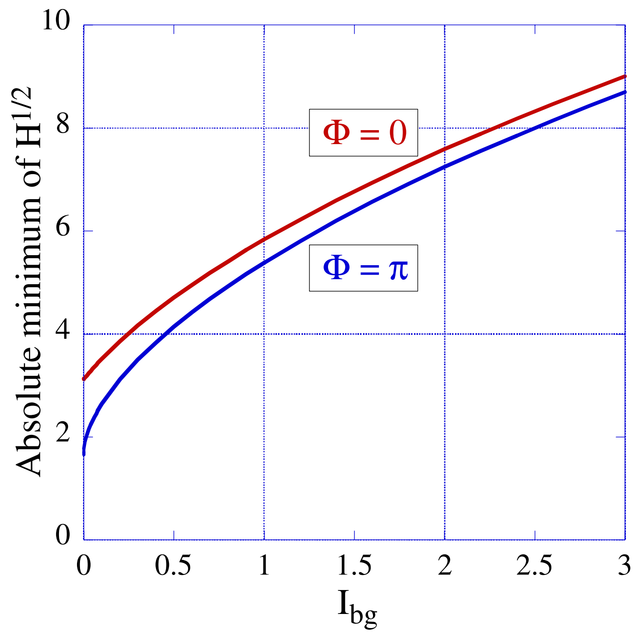

Figure 1. Absolute minimum of $H^{1/2}$ for L/l = 15

This operational optimum is obtained for the microwave level which provides the first maximum of the atomic response amplitude <u>at the probe point</u>. The fractional difference between the $H^{1/2}$ values –and consequently between the frequency stabilities– of the absolute minimum and of the operational optimum is a few percent for $\phi = \pi$ and about 10% for $\phi = 0$.

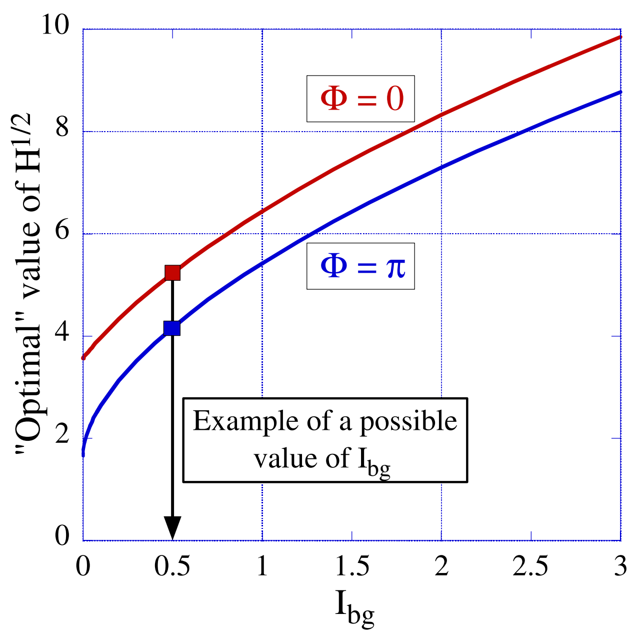

Figure 2. Optimum value of $H^{1/2}$ for L/l = 15

The optimum operational condition also concerns the modulation depth, which varies versus I_{bg}: the smaller I_{bg} parameter, the smaller ω_m if $\phi = \pi$ but the larger ω_m if $\phi = 0$.

The corresponding behaviour of the optimum $H^{1/2}$ versus I_{bg} is shown in Fig.2 and the ratio of optimum $H^{1/2}$ values for the resonators with $\phi = 0$ or π is shown in Fig 3.

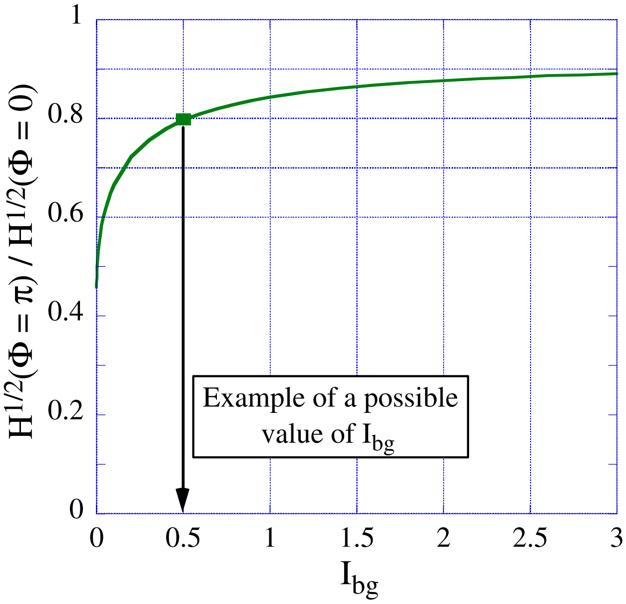

Figure 3. Ratio $H^{1/2}(\phi = \pi) / H^{1/2}(\phi = 0)$ for L/l = 15

One can observe that the improvement given by the inverted fringe ($\phi = \pi$) varies from about 10% for high values of I_{bg} corresponding to a moderate performance, to more than 30% for I_{bg} about 0.1 corresponding to a very high performance operation : the impact of the cavity with $\phi = \pi$ is specially interesting for the very high performance operation, where it enables either a better performance at the same atomic flux or a significant reduction of the cesium consumption, and consequently a longer clock lifetime, at the same performance level.

The advantage given by the inverted fringe ($\phi = \pi$) has two main origins :

- for $\phi = 0$, the transition probability is maximum (peak) at the atomic transition frequency, and is not zero at the minima (valley) of the resonance curve. This gives an additional signal which does not contribute to the determination of the reference frequency, but provides an additional and useless shot noise in the clock signal. This spurious effect does not exist with $\phi = \pi$ since the transition probability is zero at resonance, as seen in Fig.4

- Fig.5 shows the velocity distribution modulated by the transition probability $P(b, \omega)$ about the optimum interrogation parameters. We observe that with $\phi = \pi$, the transition selects slower atoms than with $\phi = 0$. This property is also favourable to the frequency stability since it provides a narrower resonance line.

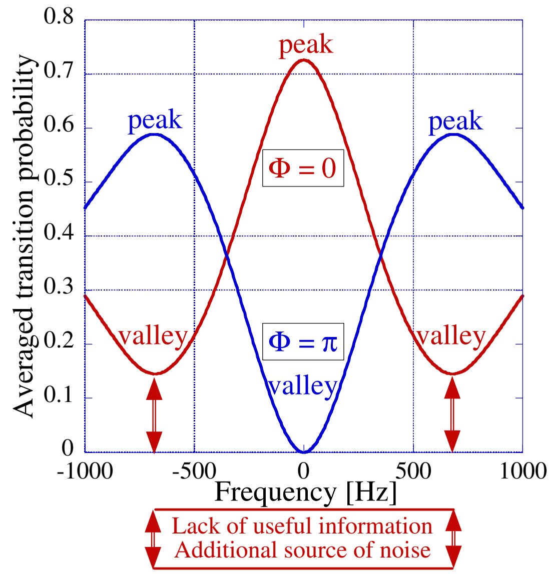

Figure 4. Example of resonance patterns with $\phi = 0$ and π.

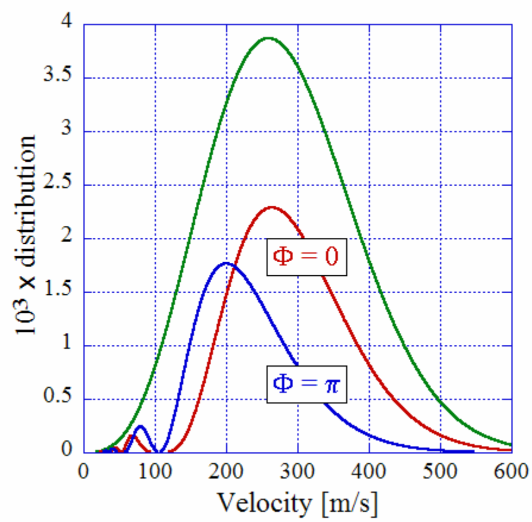

Figure 5. Velocity distribution (upper curve) modulated by the simplified Ramsey probability for $\phi = 0$ and $\phi = \pi$ (lower curves) near the optimum interrogation parameters.

IV. EXPECTED AND MEASURED FREQUENCY STABILITY

We have applied the previous analysis to the atomic resonator OSCAR developed by ON (Neuchâtel Observatory) [7][14]. The exercise consists in determining the useful elements in equation (13): T_0 from the oven temperature and the length between the arms of the cavity, I_0 from the peak to valley detected signal, F^* from (5)… and finally the optimum value of $H^{1/2}$.

NB: The result of the calculation does not give the stability of a particular clock, but the best stability that this clock can reach when its operating parameters are optimized for a particular atomic flux.

A. Experiment with D2 line (λ = 852nm)

The experimental tube OSCAR developed at ON is operated with a $\phi = 0$ cavity. In the experiment using D2 line, the atomic transition used for the optical pumping and detection is 4-4', that provides β = 2.4 photons per atom. The light source is a DFB diode laser, with a linewidth smaller than 2MHz. The temperature of the oven is 130°C, and the flux of useful atoms (peak probability ≈ 75%) in the detection zone, measured by means of a hot wire detector, is 1.57×10^{10} at/s. At this oven temperature, T_0 is 5.36×10^{-4} s.

Experimental values measured at ON with OSCAR are:

- Light collection efficiency: γ= 0.49
- Peak to valley Ramsey signal: 1.68 nA
- Noise current at the top of the Ramsey fringe: 41 fA/Hz$^{1/2}$
- Background current due to the spurious light: 2.12 nA
- Photo-detector noise: 7 fA/Hz$^{1/2}$.

The flux mentioned by ON is associated to the transition probability at the top of the Ramsey fringe, approximately 75%: consequently the I_0 value estimated from ON measurement is about 2.09×10^{10} at/s. It is also possible to estimate I_0 from the peak to valley Ramsey signal: taking into account the corresponding transition probability, this value is 2.08×10^{10} at/s, which is in perfect agreement with the previous value. The effective noise figure F^* calculated from (5) and for ON value of γ is 2.43. The estimation of I_{bg} requires the knowledge of the additional noise that contains several contributions: from ON data, we can estimate this global additional noise.

At the top of the Ramsey fringe, the atomic contribution is $2I_0P(b,\omega)\,F^*$, which gives 34.7 fA/Hz$^{1/2}$ after conversion into current fluctuations at the output of the photo-detector. The shot noise generated by the stray light background current is evaluated at 26 fA/Hz$^{1/2}$, and the overall calculated noise at the top of the Ramsey pattern is 43.9 fA/Hz$^{1/2}$. This value is larger by 7% than the measured noise 41 fA/Hz$^{1/2}$.

The main conclusion of this analysis is that there is no complementary noise source to be investigated.

The obtained I_{bg} value is 0.45. This means that the contribution of the additional noise power is 45% of the noise contribution of I_0. The optimum value (see fig2) of $H^{1/2}$ is about 5.1 for ϕ= 0, and the resulting optimum frequency stability to be expected is $\boldsymbol{\sigma_y(\tau) = 1.26\times10^{-12}\,\tau^{-1/2}}$.

The experimental value obtained at ON is $\mathbf{1.51\times10^{-12}\,\tau^{-1/2}}$, which is not so far from the expected optimum performance. We can conclude that i) the adjustments of the operational parameters (modulation depth and microwave level) are not far from the optimum values and ii) there is no other important noise source.

NB: in spite of this satisfactory experimental result, it is important to notice that the value of I_{bg} (0.45) is severely limited by the spurious light contribution (0.42).

On the basis of the same theoretical model applied to a virtual OSCARino using a cavity with $\phi = \pi$, the expected optimum frequency stability with the same atomic flux is better by 20%, leading to $1.01\times10^{-12}\,\tau^{-1/2}$. If the same improvement is applied to the experimental result, it gives $1.21\times10^{-12}\,\tau^{-1/2}$.

B. Experiments with D1 line (λ = 894nm)

Due to the lack of satisfactory DFB laser at 894nm, an extended cavity diode laser was used [7]. Its linewidth is smaller than 500kHz. The atomic transition of interest is 3-4', that provides 2.4 photons per atom. Always with an oven temperature of 130°C, the new measured characteristics are:

- Useful atomic flux (ON definition) 1.42×10^{10} at/s
- Peak to valley Ramsey signal: 1.46 nA
- Noise current at the top of the Ramsey fringe: 34 fA/Hz$^{1/2}$
- Background current (due to spurious light): 0.42 nA

The corresponding I_0 flux is 1.9×10^{10} at/s.

Following the same approach as previously, the noise contribution of the atoms is 33 fA/Hz$^{1/2}$ at the top of the resonance. The total expected noise including spurious light (11.6 fA/Hz$^{1/2}$) and photo-detector (7 fA/Hz$^{1/2}$) is 35.6 fA/Hz$^{1/2}$, to be compared with the measured value 34 fA/Hz$^{1/2}$: the agreement is excellent and it also means that there is no complementary noise contribution. The I_{bg} value obtained in this case is only 0.13, which is very low.

This excellent performance is due to the conjunction of three effects:

- the significant reduction of the spurious light, mainly due to the external cavity diode laser, which makes it possible to use the light more efficiently in the detection zone : less optical power and consequently less spurious light (contribution 0.09 instead of 0.43 in I_{bg})
- a very low noise photo-detector set-up enabled by an adequate photodiode, a very high transimpedance and a very low noise operational amplifier (contribution 0.04 in I_{bg})
- a very high atomic flux I_0

With I_{bg} = 0.13, the optimum frequency stability that can be expected is 1.07×10^{-12} (ϕ = 0) to be compared with 1.14×10^{-12} measured at ON: the agreement is very satisfactory.

On the basis of the same theoretical model applied to a virtual OSCAR using a cavity with $\phi = \pi$, the expected optimum frequency stability with the same atomic flux is better by 33%, leading to $7.2\times10^{-13}\,\tau^{-1/2}$.

V. CONCLUSION

The predictions of the theoretical model developed by SYRTE and the experimental results obtained at Observatoire de Neuchâtel are in very good agreement. Several redundant results are precisely confirmed, and this ensemble is quite selfconsistent. Consequently we are very confident in the theoretical results concerning the configuration with $\phi = \pi$: it will enable either better frequency stability at the same cesium consumption and lifetime, or the same frequency stability with reduced cesium consumption and consequently a significantly larger lifetime, which is especially interesting for space applications.

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